

SN - 353

V Semester B.A./B.Sc. Examination, November/December 2017
(Semester Scheme) (CBCS) (2016 - 17 & Onwards)
(Fresh + Repeaters)
MATHEMATICS - V

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all questions.

PART - A

Answer any five questions :

(5x2=10)

- 1 a) In a ring $(R, +, \cdot)$ prove that $\forall a, b, c \in R, a \cdot (b - c) = a \cdot b - a \cdot c$.
- b) Show that the set of even integers is not an ideal of the ring of rational numbers.
- c) Prove that every field is a principal ideal ring.
- d) If $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$, show that \vec{F} is irrotational.
- e) Find the maximum directional derivative of $x\sin z - y\cos z$ at $(0, 0, 0)$.
- f) Prove that $E\nabla = \nabla E = \Delta$.
- g) Construct the Newton's divided difference table for the following data :

x	4	7	9	12
f(x)	-43	83	327	1053

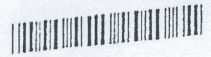
- h) Using Trapezoidal rule to evaluate $\int_0^1 \frac{dx}{1+x}$ where

x	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
y = f(x)	1	0.8571	0.75	0.6667	0.6	0.5455	0.5

P.T.O.

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PART - B

Answer **two full** questions :

(2×10=20)

2. a) Prove that the set $R = \{0, 1, 2, 3, 4, 5\}$ is a commutative ring with respect to ' \oplus_6 ' and ' \otimes_6 ' as the two compositions.
b) Prove that a ring R is without zero divisors if and only if the cancellation laws hold in R

OR

3. a) Show that an ideal S of the ring of integers $(\mathbb{Z}, +, \cdot)$ is maximal if and only if S is generated by some prime integer.
b) Prove that a commutative ring with unity is a field if it has no proper ideals.
4. a) If R is a ring and $a \in R$, let $I = \{x \in R / ax = 0\}$ prove that I is a right ideal of R .
b) If $f : R \rightarrow R'$ be a homomorphism with kernel K , then prove that f is one-one if and only if $K = \{0\}$.

OR

5. a) Let $R = R' = \mathbb{C}$ be the field of complex numbers. Let $f : R \rightarrow R'$ be defined by $f(z) = \bar{z}$ where \bar{z} is the complex conjugate of z , show that f is an isomorphism.
b) Prove that every homomorphic image of a ring R is isomorphic to some residue class (quotient) ring thereof.

PART - C

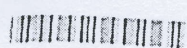
Answer **two full** questions :

(2×10=20)

6. a) Prove that $\nabla^2(f(r)) = f''(r) + \frac{2}{r} f'(r)$, where $r^2 = x^2 + y^2 + z^2$.

- b) Find the unit normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1, 2, -1)$.

OR



7. a) Show that $\text{Curl} [\vec{r} \times (\vec{a} \times \vec{r})] = 3\vec{r} \times \vec{a}$ where \vec{a} is constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

b) If the vector $\vec{F} = (3x + 3y + 4z)\hat{i} + (x - 4y + 3z)\hat{j} + (3x + 2y - z)\hat{k}$ is solenoidal, find 'a'.

8. a) Prove that $\nabla^2 \left(\frac{1}{r} \right) = 0$, where $r^2 = x^2 + y^2 + z^2$.

b) If $\vec{F} = \nabla (2x^3 y^2 z^4)$, find $\text{Curl } \vec{F}$ and hence verify that $\text{Curl} (\nabla \phi) = 0$.
OR

9. a) If ϕ is a scalar point function and \vec{F} is a vector point function, prove that

$$\text{div} (\phi \vec{F}) = \phi \text{div } \vec{F} + \text{grad } \phi \cdot \vec{F}$$

b) Find $\text{Curl} (\text{Curl } \vec{F})$ if $\vec{F} = x^2 y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$.

PART - D

Answer two full questions :

(2×10=20)

10. a) Use the method of separation of symbols to prove that

$$u_0 + u_1 x + u_2 x^2 + \dots \text{ to } \infty$$

$$= \frac{u_0}{1-x} + \frac{x \Delta u_0}{(1-x)^2} + \frac{x^2 \Delta^2 u_0}{(1-x)^3} + \dots \text{ to } \infty.$$

b) i) Evaluate $\Delta^{10} [(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)]$.

ii) Express $f(x) = 3x^3 + x^2 + x + 1$ as a factorial polynomial (taking $h = 1$).

OR



11. a) Find a second degree polynomial which takes the following data :

x	1	2	3	4
f(x)	-1	-1	1	5

- b) Find $f(1.9)$ from the following table :

x	1	1.4	1.8	2.2
f(x)	2.49	4.82	5.96	6.5

12. a) Using Lagrange's interpolation formula find $f(6)$ for the following data :

x	2	5	7	10	12
f(x)	18	180	448	1210	2028

- b) Using Simpson's $\frac{3}{8}$ rule evaluate $\int_0^{0.6} e^{-x^2} dx$ by taking 6 sub intervals.

OR

13. a) Following is the table of the normal weights of babies during the first few months of life.

Age in months	2	5	8	10	12
Weight in kgs	4.4	6.2	6.7	7.5	8.7

Estimate the weight of a baby of 7 months old using Newton's divided difference table.

- b) Obtain an approximate value of $\int_0^6 \frac{dx}{1+x^2}$ by Simpson's $\frac{1}{3}$ rule.